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The effect of quasi-periodicity on the resonant tunneling lifetimes of states in electrically biased semiconductor superlattices

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Abstract

The tunneling lifetimes of quasi-resonant states for electrons in various kinds of generalized Fibonacci and generalized Thue–Morse GaAs–$Al_xGa_{1-x}As$ superlattices have been evaluated numerically under variable dc bias conditions. All the quasi-periodic systems have been framed using the generalized block model. The variation of the lifetime at different quasi-resonant levels with respect to the external dc field undergoes a remarkable change due to the effect of quasi-periodicity. The occurrence of a minimum value of the average lifetime and its dependence on quasi-periodicity have been analyzed. It is shown that the low-order quasi-periodicity in the case of the generalized Fibonacci superlattice and the high-order quasi-periodicity for the generalized Thue–Morse superlattice hold promise for potential device applications. The impact of an increase in the number of barriers on the tunneling lifetime has also been studied exhaustively.

1. Introduction

The problem of resonant tunneling phenomena in semiconductor multibarrier heterostructures has long been of great interest, since the pioneering work of Tsu and Esaki [1] in the early 1970s. The widely discussed features such as the transmission coefficient, resonant tunneling energy levels, density of states, tunneling lifetime and traversal time which provide signatures of resonant tunneling have profound importance in potential device applications. Although the fundamental properties of tunneling are well understood conceptually, the significance still needs to be unraveled for some of them. For example, the well known resonant tunneling lifetime, which is of great importance in explaining the carrier transport in one-dimensional superlattices, remains very controversial and debatable. A thorough knowledge of the aforesaid lifetime is very much required for accuracy and appropriate applicability for extracting the best performance from high-speed electronic and optoelectronic devices like lasers, modulators, photo-detectors and signal processing devices. Most of the previous theoretical works on resonant tunneling lifetimes in double-barrier systems [2–7], periodic multibarrier systems [8] and periodic superlattices (PSLs) [9–13] are limited to the field-free condition. The appearance of a special kind of minima in tunneling lifetime spectra and their explanation based on the infinite Kronig–Penney model are reported by Khan *et al* [8] in the case of unbiased periodic multibarrier systems with $N > 3$ (N being the number of barriers). When such a system is subjected to a dc field, the former resonant tunneling energy states in the unbiased condition get Stark shifted giving rise to Stark states. Though resonant tunneling corresponds to unit transmission coefficient in the field-free case, the transmittance for some

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of the Stark states becomes less than unity. Tunneling states of this type are referred to as quasi-resonant tunneling states and the lifetime of these states is called the quasi-resonant tunneling lifetime (QRTL). At low and medium field strength, the separations between the consecutive states are not the same. But when the applied electric field (E) satisfies the inequality [14]

$$eEd > \tfrac{1}{2}\Delta\varepsilon_{\mathrm{m}}, \qquad (1.1)$$

d being the periodicity of the superlattice and $\Delta\varepsilon_{\mathrm{m}}$ the maximum level separation in a miniband of the PSL, the Stark states start to be spaced equally, forming a ladder around the center of the miniband. This ladder-like energy spectrum shows the onset of the partial Wannier–Stark ladder (WSL). More and more states join the ladder with increase of the field strength. Recently, the WSL has been exploited in tunable quantum cascade lasers and for realizing this the QRTL of these states needs to be determined. We have reported [15] anomalous behavior of the carrier lifetime for the Stark states in the case of periodic superlattices, where the carrier transport has been explained on the basis of the WSL. The minimum value of the average QRTL that corresponds to the selection of carriers of a maximum velocity through the system and the corresponding electric field strength have been defined as the characteristic parameters for PSLs consisting of a particular number of barriers.

Although exhaustive studies on the resonant tunneling lifetime in the case of PSLs have been performed both experimentally and theoretically, the research on the tunneling lifetime in aperiodic systems is still gaining momentum. The discovery of quasi-crystals has accelerated the study of aperiodic superlattices (ASLs) arranged according to the standard generalized Thue–Morse and generalized Fibonacci sequences. The quasi-periodic system, an intermediate between a perfectly ordered system and a disordered system, deserves special attention in the study of unique electronic and optical properties [16] different from the above two extreme cases. Ideal ASL shows a highly fragmented and fractal-like electronic spectrum with self-similar patterns, with the presence of localized, extended and critical states [17, 18]. A lot of theoretical works regarding the band structures [19], transmission properties [20, 21], density of states [22], localization of wavefunctions [23–25], trace maps and Landauer resistances [26–28] etc in ASL have been reported both in the absence and in the presence of externally applied dc electric fields. Recently, we have calculated the current density of the Fibonacci superlattice [29] and have shown that such ASL can behave as band-pass or band-eliminator semiconductor devices. Moreover, the degree of aperiodicity has a profound effect in negative differential conductivity regions, which must be considered prior to device applications. Although the generalized Fibonacci sequence is in frequent use for studying the transport properties in ASLs, the use of the generalized Thue–Morse sequence is quite limited both in theory and experiment. Further, little attention [21] has been paid to the study of the QRTL in the case of aperiodic systems like the generalized Fibonacci superlattice (GFSL) and generalized Thue–Morse superlattice (GTSL) in the presence of a homogeneous electric field and the present intention is to report on an effort in this direction.

In the following study we have presented a detailed analysis of the QRTL in the case of aperiodic superlattices (e.g., GFSL, GTSL) as well as for a periodic one, both in the presence and in the absence of external dc fields. For the calculation of the QRTL across multibarrier systems, our theoretical model is based on the transfer-matrix formalism using an exact Airy function approach and a search technique [13, 15]. The beauty of this method is that it is straightforward and gives the same order of accuracy as those shown in other works [30–33]. Moreover, an account of the mean or average value of the QRTL has also been given for the different structures mentioned above.

2. Models for different superlattices

In the present theoretical model, both the periodic and generalized aperiodic superlattice structures have been grown along the z-axis starting from two basic building blocks A and B [23]. Here the block A (B) consists of a rectangular quantum well of thickness a and a rectangular barrier of thickness b (b').

The present block model of the PSL has been generated by iterating A and B alternately, so that the tth generation of the system is given by $S_t = t[\mathrm{AB}]$, where t stands for the number of repetitions. This model is different from the typical one [15] in the sense that here two barriers (instead of one) of different widths separated by a well appear periodically.

The tth generation of any kind of generalized Fibonacci sequence [34] has been framed using the following recursion rule: $S_t = [m[S_{t-1}] \cdot n[S_{t-2}]]$, for $t > 0$. Here m and n give the number of repetitions of the associated generation and the centered dot denotes concatenation of strings. A given pair of values (m, n) represents a particular kind of generalized Fibonacci sequence, e.g., ($m = 1$, $n = 1$), ($m = 2$, $n = 1$) and ($m = 1$, $n = 3$) are the first, second and third kinds, respectively. The initial conditions for the generation of any kinds of such sequences are chosen as $S_{-1} = \mathrm{B}$ and $S_0 = \mathrm{A}$.

The different generalized Thue–Morse sequences [27] have been obtained using the iteration formula $S_t = [m[S_{t-1}] \cdot n[\bar{S}_{t-1}]]$, for $t > 0$. $\bar{S}_{t-1}$ is the complement of S_{t-1} obtained by interchanging A and B. In this case the pairs of values (1, 1), (2, 2) and (1, 3) for (m, n) are referred to as first, second and third kinds of generalized Thue–Morse sequences, respectively. Here the initial condition is chosen as $S_0 = \{\mathrm{A}, \mathrm{B}\}$ for all kinds.

For both the blocks A and B, a small gap material GaAs is taken as the well region, and the well width, a, consists of five cells, whereas a large gap material $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ acts as the barriers of variable thicknesses (b and b') consisting of five and six cells for A and B blocks, respectively. The first few generations of the three kinds of generalized Fibonacci and Thue–Morse sequences are presented in tables 1 and 2. From these tables it is understood that in both the sequences the quasi-periodicity increases more in a higher kind than in a lower kind [28].

Table 1. A few initial generations of generalized Fibonacci sequences for different values of m and n with initial conditions $S_{-1} = \mathrm{B}$ and $S_0 = \mathrm{A}$.

t	S_t	$S_t = [1[S_{t-1}] \cdot 1[S_{t-2}]]$	S_t	$S_t = [2[S_{t-1}] \cdot 1[S_{t-2}]]$	S_t	$S_t = [1[S_{t-1}] \cdot 3[S_{t-2}]]$
1	S_1	AB	S_1	AAB	S_1	ABBB
2	S_2	ABA	S_2	AABAABA	S_2	ABBBAAA
3	S_3	ABAAB				
4	S_4	ABAABABA				
5	S_5	ABAABABAABAAB				

Table 2. A few initial generations of generalized Thue–Morse sequences for different values of m and n with initial condition $S_0 = \{\mathrm{A, B}\}$.

t	S_t	$S_t = [1[S_{t-1}] \cdot 1[\bar{S}_{t-1}]]$	S_t	$S_t = [2[S_{t-1}] \cdot 2[\bar{S}_{t-1}]]$	S_t	$S_t = [1[S_{t-1}] \cdot 3[\bar{S}_{t-1}]]$
1	S_1	ABBA	S_1	ABABBABA	S_1	ABBABABA
2	S_2	ABBABAAB				
3	S_3	ABBABAABBAABABBA				

3. A brief formulation for computing the QRTL

To deal with the present problem one has to consider the one-dimensional time-independent Schrödinger equation for the electron in the potential $V(z)$, which appears as

$$-\frac{\hbar^2}{2m^*_{1,2}}\frac{\mathrm{d}^2\psi^{1,2}}{\mathrm{d}z^2} + V(z)\psi^{1,2} = \varepsilon\psi^{1,2} \tag{3.1}$$

where the subscripts and superscripts, 1 and 2, stand for the corresponding parameters in the well and barrier regions, respectively. Here, ε and m^* represent the incident energy and effective mass of the electron. The potential energy profile $V(z)$ for the superlattice with a homogeneous electric field E applied along the growth direction (z-axis) between $z = 0$ and l is represented by

$$V(z) = \begin{cases} V_0 - eEz & \text{for } z_{n\mathrm{L}} \leqslant z \leqslant z_{n\mathrm{R}} \\ -eEz & \text{otherwise,} \end{cases} \tag{3.2}$$

where V_0 is the potential barrier height and e the electronic charge, $z_{n\mathrm{L}}$ and $z_{n\mathrm{R}}$ being the left and right boundaries of the nth barrier, respectively.

Using exact Airy function formalism, (3.1) can be solved for the wavefunctions in the well and barrier regions. Then applying effective-mass-dependent boundary conditions [35], one can find the transfer matrix ($[M_n]$) that correlates the amplitudes of the wavefunctions to the right (A_{2n+1} and B_{2n+1}) of the nth barrier with those to the left (A_{2n-1} and B_{2n-1}) of the same barrier, given by

$$\begin{bmatrix} A_{2n+1} \\ B_{2n+1} \end{bmatrix} = [M_n] \begin{bmatrix} A_{2n-1} \\ B_{2n-1} \end{bmatrix}. \tag{3.3}$$

Ultimately, the transfer matrix that correlates the amplitudes of the wavefunctions for $z < 0$ (A_0 and B_0) and $z > l$ (A_F and B_F) takes the form

$$\begin{bmatrix} A_F \\ B_F \end{bmatrix} = [F]\prod_{n=1}^{N}[M_n][I]\begin{bmatrix} A_0 \\ B_0 \end{bmatrix} = [W_N]\begin{bmatrix} A_0 \\ B_0 \end{bmatrix} \tag{3.4}$$

where $[W_N] = [F]\prod_{n=1}^{N}[M_n][I]$ and N is the number of barriers. Here, the transfer matrices designated by $[I]$ and $[F]$, respectively, correlate the amplitudes of the wavefunctions to the left and right of $z = 0$ and l, respectively.

Finally, the coefficient of transmission (τ_c) across the N-barrier superlattice [29, 36] can be given by

$$\tau_\mathrm{c} = \frac{|A_F|^2}{|A_0|^2} = \left|\frac{\det[W_N]}{(W_N)_{22}}\right|^2. \tag{3.5}$$

Using the relation (3.5) we have, first of all, computed τ_c for a range of energies ($\varepsilon < V_0$) for the incident electron at different values of the dc field. From the transmission spectrum for a particular value of the field, the energy (ε_m) corresponding to the quasi-resonant transmission peaks and the energies on both sides of the peak corresponding to τ_c at half of the transmission maxima have been obtained numerically. From the difference of the two energies corresponding to half-maxima one can find the halfwidth at half-maximum ($\Delta\varepsilon_\mathrm{m}$) corresponding to the quasi-resonant energy, ε_m.

At the end, these numerical values of $\Delta\varepsilon_\mathrm{m}$ are used to calculate the QRTL (τ) on the basis of an energy uncertainty relation given by [2]:

$$\tau = \frac{\hbar}{2\Delta\varepsilon_\mathrm{m}}. \tag{3.6}$$

4. Numerical analysis

In the present work the comparative analysis of the QRTL, among the different multibarrier systems, has been performed with the incident electron energies in the below-barrier condition ($\varepsilon < V_0$). Here, the basic system is considered as a GaAs–$\mathrm{Al_{0.3}Ga_{0.7}As}$ superlattice, choosing the conduction band discontinuity or the barrier height (V_0) to be 370.1 meV [37]. It may be mentioned that in the below-barrier condition, only one allowed energy miniband exists for all the systems under investigation. Here each of the aperiodic systems has been studied taking three kinds with constant number of barriers. The electron effective masses in the two host materials are $m_1^* = 0.065m_0$ (in the well region) and $m_2^* = 0.0919m_0$ (in the barrier region), m_0 being the free electron mass. The lattice constants, i.e., cell widths for the well and barrier materials, are considered as 5.6533 Å

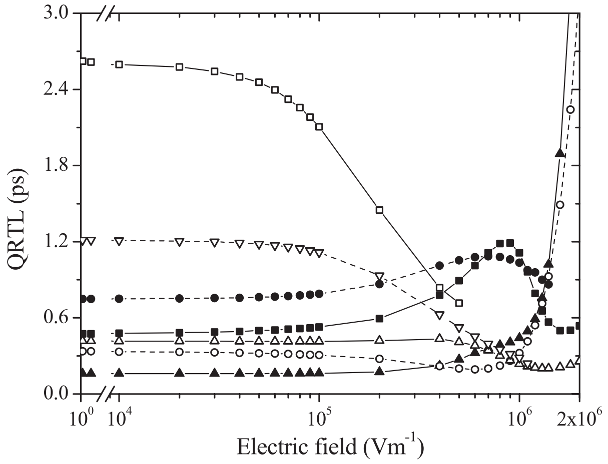

Figure 1. Plot of the QRTL (τ) versus the applied dc electric field (E) for the block PSL using a GaAs–$Al_{0.3}Ga_{0.7}As$ system with $N = 8$. L1 (open square, solid line); L2 (solid circle, dashed line); L3 (solid square, solid line); L4 (solid triangle, solid line); L5 (open circle, dashed line); L6 (open triangle, solid line); L7 (open inverted triangle, dashed line): L1–L7 correspond to the quasi-resonant energy states (e.g., 1 for the minimum energy state) in the miniband.

and 5.5564 Å respectively. The homogeneous electric field applied across the structure has been varied from 1 V m^{-1} to 2 MV m^{-1}. For the zero-field case, the calculation based on the Airy function is not possible, as the argument of the function contains the field parameters in the denominator. Our present numerical results for the field 1 V m^{-1} become identical (up to three significant decimal digits) to the results of analytic calculation for unbiased conditions by Nanda *et al* [13]. So, for all practical purposes, the electric field 1 V m^{-1} can be treated in our model as the unbiased condition (field-free case).

5. Results and discussion

The quasi-resonant tunneling lifetime is numerically computed on the basis of (3.6) for GaAs–$Al_{0.3}Ga_{0.7}As$ superlattices under a homogeneous electric field varying from 1 V m^{-1} to 2 MV m^{-1}, with the blocks (A and B) arranged periodically and quasi-periodically. For the comparison of QRTLs among the block periodic and various kinds of aperiodic superlattices (generalized Fibonacci and Thue–Morse), we have selected $N = 8$ (figures 1–3). In all the cases the number of peaks in the miniband (under below-barrier conditions) has been found to be seven in unbiased condition. This feature is analogous to the occurrence of $(N-1)$ resonant energy levels in the case of a field-free N-barrier periodic system [8, 11]. It may be pointed out that the present approach is more general than our previous two models [15, 29] in the sense that here the periodic and the aperiodic superlattices have been generated with the blocks A and B instead of the simple wells and barriers.

Figure 1 shows the variation in QRTL for all seven levels in the block PSL with varying dc fields. It may be noted from the figure that the QRTL for all levels remains almost constant up to $E = 30$ kV m^{-1}. This is quite justified since at this field strength the Stark shift, being of the order of 10^{-4} eV, is very small compared to the field-free miniband

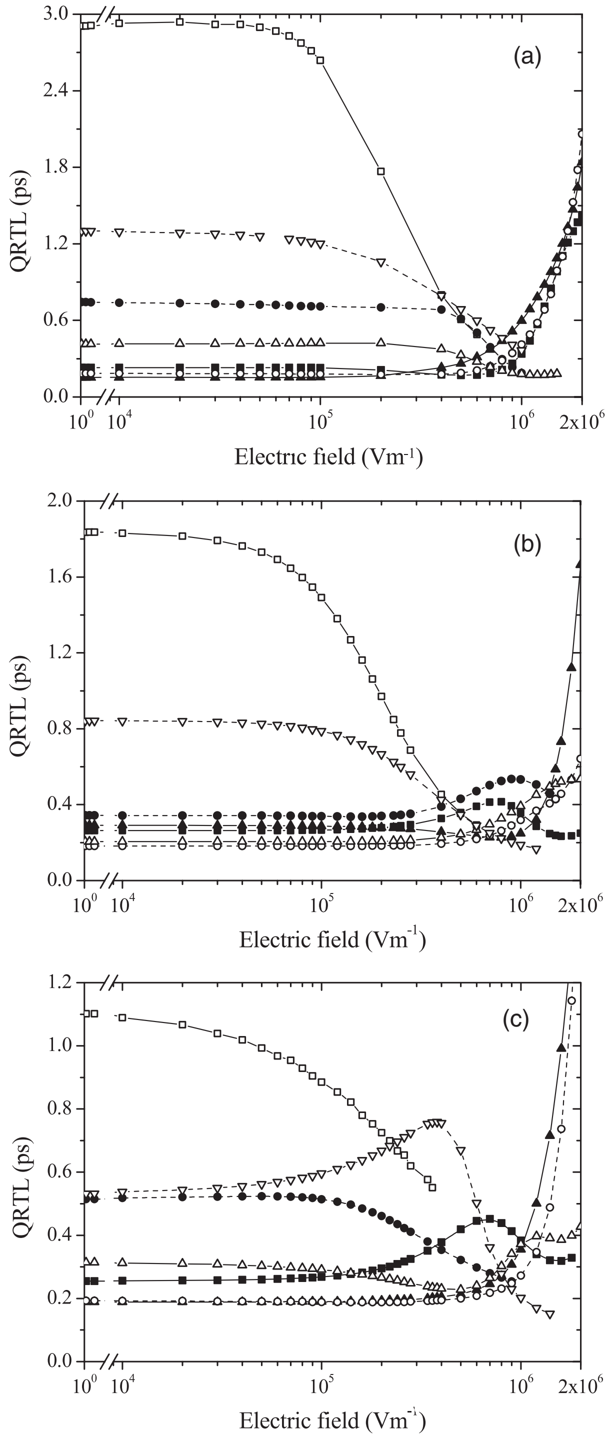

Figure 2. The same as figure 1 for (a) the first kind ($m = 1$, $n = 1$), (b) the second kind ($m = 2$, $n = 1$) and (c) the third kind ($m = 1$, $n = 3$) of GFSLs.

level separation. So the wavefunctions corresponding to these states remain extended in nature and the coupling between the wells suffers no significant change. Thus the tunneling lifetime and hence the velocity remain almost unchanged up to this field strength. It is also clear from the figure that the effect of applied bias is first substantially realized by the outermost wells from the applied field $E \sim 100$ kV m^{-1} and onwards. So the transmission coefficient and the tunneling

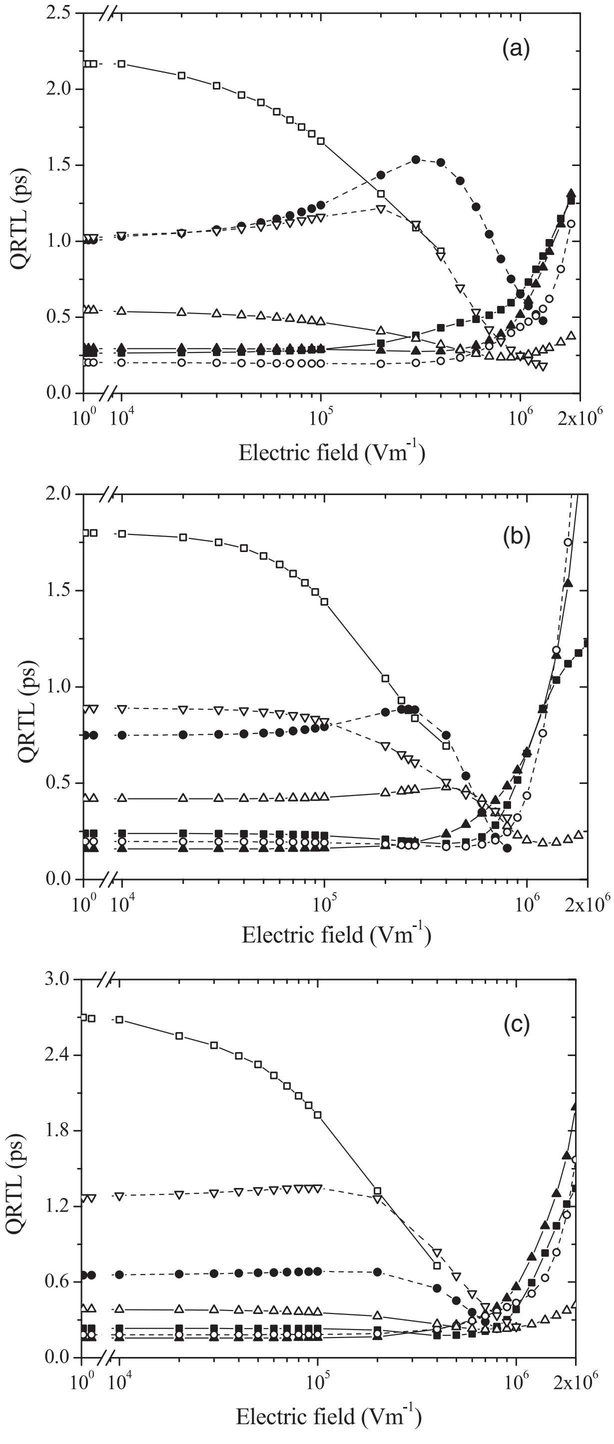

Figure 3. The same as figure 1 for (a) the first kind ($m = 1, n = 1$), (b) the second kind ($m = 2, n = 2$) and (c) the third kind ($m = 1, n = 3$) of GTSLs.

lifetime for electrons in the band edge states suffer significant changes from this field strength. In this region, the magnitude of the QRTL decreases as one proceeds towards the center of the miniband. This suggests that an electron with quasi-resonant tunneling energy, ε_m, corresponding to the middle of the band would tunnel out faster than the levels with other values of ε_m. This result is analogous to our earlier findings in the case of a typical periodic superlattice [11, 15]. The lower lifetime of the mid-band states is the consequence of the larger group velocity of electrons in these states. For the typical PSL, the lifetime of each band edge state is found to decrease with increase in the field. But in the present block PSL, although the lifetime for the states far away from the middle decreases with field and dies out quickly, the near mid-band states in the lower half of the miniband show a prominent crest where the QRTL reaches a maximum value and then decreases gradually. This type of crest with a region of positive differential lifetime or, in other words, negative differential velocity is absent in periodic superlattices for near mid-band states. The discrepancy arises due to incorporation of quasi-periodicity to some extent in the block PSL. At very high fields ($E > 1$ MV m^{-1}), like in the periodic case [15], only the mid-band states survive, so tunneling of electrons takes place through the mid-band only and the corresponding lifetime increases linearly with increasing field strength. This feature can be comprehended as follows. Beyond a certain limit of the applied field satisfying (1.1) the mid-band states start forming a partial WSL, consequently increasing the tunneling lifetime and hence decreasing the tunneling velocity of the carriers associated with these levels. The band edge states which do not join the WSL follow the general pattern of increase of the group velocity and hence decrease in carrier lifetime with increase of the field. One interesting feature is also to be noted that, in typical PSL, there exists a characteristic electric field where all the states have equal value of tunneling lifetime, i.e., the electron in each state tunnels with the same group velocity at that characteristic field [15]. But in the present PSL (block) such synchronization of carrier velocities for all the states has not been achieved.

Figures 2(a)–(c) show the variation of the QRTL with the applied field in all the quasi-resonant energy levels for the fourth, second and second sequences of first, second and third kinds of generalized Fibonacci superlattices, respectively. To make the total length or total barrier numbers of the superlattice almost identical ($N = 8$), an additional block A for the second and third kinds of GFSLs has been appended. Here the initial variation of the QRTL shows the same features as those in the case of the present block PSL. From figure 2(a) it is found that the magnitude of the lifetime at low fields, as compared to the periodic case, shows an increase for the band edge states but a decrease for the mid-band states. This indicates that due to quasi-periodicity the inner wells are more strongly coupled and the reverse is the case for the outer wells. Here, the near mid-band states do not show any crest, as observed in the block periodic case. Thus as far as the variation of the QRTL is concerned, the first kind of GFSL shows a closer resemblance to the typical PSL than to the block PSL.

From figures 2(a)–(c) we also find that there is an overall decrease in magnitude of lifetime for the band edge states as one proceeds towards the higher kinds of GFSLs. This can be attributed to the effect of increase in aperiodicity. In GFSLs of higher kinds, unlike the typical PSL and the block PSL, the electrons move with the maximum velocity in the near mid-band state rather than the mid-band state. Here the electrons in the near mid-band state 'prefer' to move with the maximum velocity unlike the mid-band state case. The three kinds of GFSLs also show a remarkable change in the variation of the QRTL at around $E = 600$ kV m^{-1}. Some of the near band edge states in the higher kinds of GFSLs show a crest where the tunneling lifetime reaches a maximum value and then decreases. The region of positive differential lifetime is absent in PSLs for near band edge states whereas it becomes prominent for higher kinds of GFSLs. For the mid-band states this region appears at a field strength $E > 1$ MV m^{-1}.

Figures 3(a)–(c) depict the variation of the QRTL with field for three different kinds of generalized Thue–Morse superlattices having the same numbers of barriers (eight). Here also the lifetime remains almost constant, like the previous ones, up to the field $E = 30$ kV m^{-1}. For a given state (except for the band edges), we notice that the QRTL at low fields decreases as one approaches from the lower to the higher kinds of GTSLs. Like for the GFSLs of higher kinds, some of the states in the GTSLs of lower kinds show a positive differential lifetime region before their disappearance. The peak height in this region for higher fields also decreases (near band edge states) for higher kinds and tends to vanish. Thus the degree of quasi-periodicity in the cases of GFSLs and GTSLs affects in reverse order the middle and near edge states of the miniband.

Since the magnitude of the QRTL is different for different states in a miniband, we have finally calculated the mean or average value of the QRTL associated with a miniband for different superlattice structures. Figures 4(a) and (b) represent the variation of the average QRTL against the applied field for the different kinds of GFSLs and GTSLs, respectively, along with the corresponding variation for the block PSLs. From these two figures we notice that up to low field strength ($E < 30$ kV m^{-1}) the average lifetime remains almost constant as explained previously and then it decreases with increase of the field strength. After reaching a minimum value, it finally rises abruptly with increasing field. This general variation occurs for all systems considered. Looking at figure 4(a) we find that the first kind of GFSL has lower average group velocity than the other two kinds and it is almost equal to that of the present PSL (block) up to the field strength, $E \sim 100$ kV m^{-1}. This shows that the greater quasi-periodicity enhances the average coupling between the barriers. With increase in the quasi-periodicity the net effective potential decreases, thereby increasing the average tunneling velocity of electrons in the aperiodic superlattices. Thus a fractal-like energy spectrum causes the electrons to move with greater velocity within a miniband. The difference in average QRTL among the different kinds of GFSLs becomes minimal around the field $E \sim 800$ kV m^{-1}, i.e., at this field the different quasi-periodic systems come close in effect as far as the average tunneling lifetime is concerned. In the case of GTSLs such resemblance has been found for higher kinds. It is interesting to note that the minimum value of the average lifetime in almost all cases lies around 0.3 ps. This value has been found to be in good quantitative agreement with that for the PSL [15] with $N = 8$ (not shown). In view of the above discussion, we may infer that the fractal-like energy spectrum of ASLs approaches to the PSL spectrum within a certain field range (700 kV m^{-1}–1 MV m^{-1}). Further, the variation of the average QRTL for the first kind of GFSL shows very close proximity to that for the third kind of GTSL, though they are two extreme cases of the two different ASLs under consideration. The variation in average lifetime with the applied field is also maximal in these two cases. Since the sharp variation of the QRTL (average) near its minima is exploited for device applications, we can therefore conclude that the first kind of GFSL and the third kind of GTSL are most suitable for the design of resonant tunneling devices. As higher kinds of ASLs being more aperiodic in nature, the above findings show that the periodicity of the GFSL and the quasi-periodicity of the GTSL promise well for device applications.

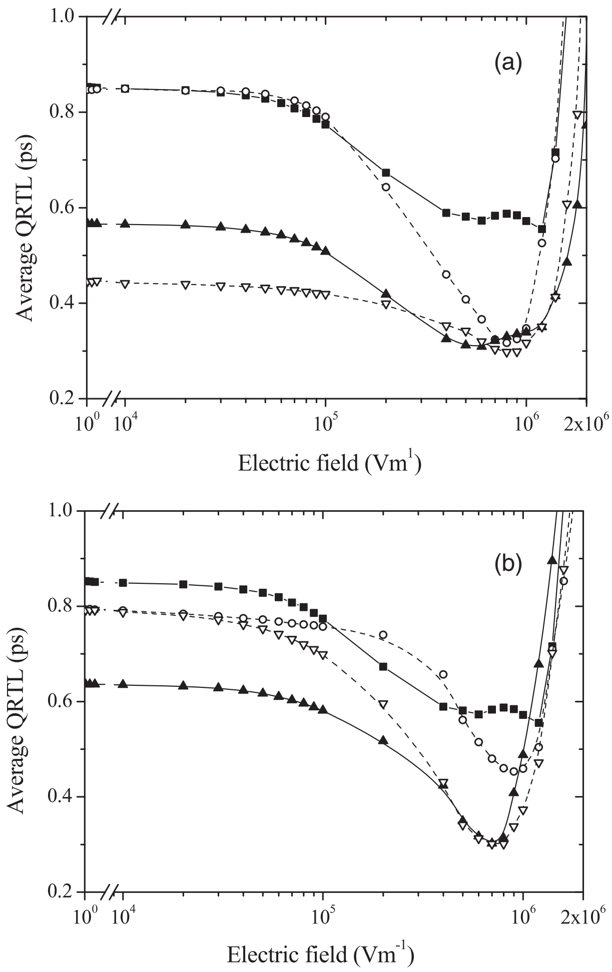

Figure 4. Plot of the average QRTL versus the applied dc electric field (E) using a GaAs–$Al_{0.3}Ga_{0.7}As$ system with $N = 8$. Curves are given for the first kinds (open circle, dashed line), the second kinds (solid triangle, solid line) and the third kinds (open inverted triangle, dashed line) of (a) GFSLs (b) GTSLs. The solid square and solid line represent the curve for the block PSL.

Figures 5(a) and (b) show the variation in average QRTL for three consecutive generations of the first kind in the case of the GFSL and the GTSL, respectively. The corresponding barrier numbers for GFSLs are chosen as $N = 5$, 8 and 13, whereas they are 4, 8, and 16 for the GTSLs. Both figures show that for all field strengths the average QRTL increases with increase in the barrier number and this is natural, due to the increase in effective length of the superlattices. All the curves show a minimum which corresponds to the maximum average velocity of electrons attainable in the superlattice for

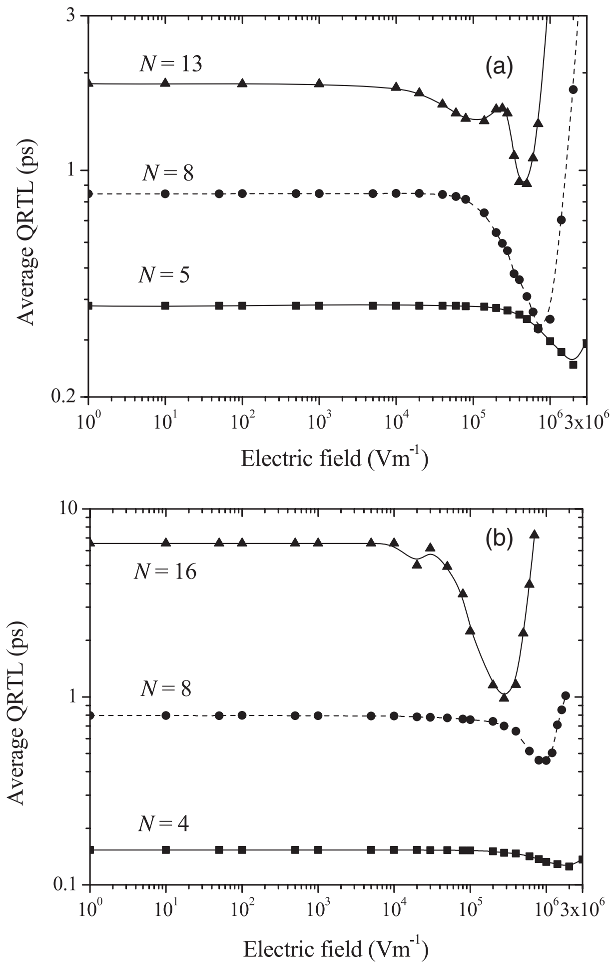

Figure 5. Plot of the average QRTL versus the applied dc electric field (E) for the first kinds of (a) GFSLs and (b) GTSLs using a GaAs–$Al_{0.3}Ga_{0.7}$As system, with different numbers of barriers (N).

a given number of barriers. The increase in barrier number causes the oscillation in the average lifetime at high fields and the appearance of a sharper minimum. The higher the value of N for a particular type of ASL, the lower the value of the field strength at which the minima occur. Although the positive differential lifetime region at moderate fields disappears in the average result, this region, which corresponds to negative differential velocity for higher fields, becomes sharper with increase in the barrier number. The existence of minima in the average lifetime curve and sharp positive differential lifetime regions may help experimentalists to select the field strength, the type and the number of barriers in the ASL for optimum performance of resonant tunneling devices.

6. Conclusion

An attempt has been made to explore the impact of quasi-periodicity on the resonant tunneling lifetime in electrically biased semiconductor superlattices. We have addressed various important issues regarding quasi-periodicity to explore new directions for analyzing high-speed semiconductor devices. The use of block periodic and aperiodic superlattices is one of the aspects providing present motivation. It has been established that the block PSL is to some extent quasi-periodic, unlike the PSL. The incorporation of quasi-periodicity modifies the internal potential of the system in the presence of external dc fields and gives rise to extra regions of positive differential lifetime for some of the quasi-resonant energy levels. In the case of generalized Fibonacci and Thue–Morse superlattices the degree of quasi-periodicity affects in reverse order the middle and near edge states of the miniband. But the electric field tunes the different aperiodic systems in such a way that the average tunneling lifetimes become almost identical at a certain field regime irrespective of aperiodicity. Thus a particular range of applied field causes the fractal-like energy spectrum to disappear. The rate of increase of the average group velocity (near its maxima) with the applied field has been found to be strongly dependent on the quasi-periodicity. This observation is an interesting outcome of the present theoretical investigation into ballistic transport of carriers through an ASL. We have finally come to the conclusion that the lower kind of GFSL and the higher kind of GTSL are the most suitable for resonant tunneling device applications. In particular, the presence of sharp minima for higher number of barriers is very important in the context of analyzing the device properties of ultrahigh-speed electronic and optoelectronic devices.

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